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Old and new wavenumber-explicit estimates for the norms and condition numbers of boundary integral operators in acoustic scattering

Boundary integral equations (BIEs) and associated representations of solutions as layer potentials are key tools for the analysis of problems of time-harmonic obstacle scattering and for their efficient numerical solution when the medium of propagation has constant or piecewise constant coefficients. We focus, for brevity, on the specific case of scattering by a sound soft obstacle, where the total field vanishes on the obstacle Γ and the medium has constant wave speed in the complement of Γ, so that we solve the Helmholtz equation with constant wavenumber $k > 0$. We recall the standard 1st and 2nd kind BIE formulations in the case when Γ is the boundary of a bounded Lipschitz domain. We also recall the standard 1st kind formulation when Γ is a thin screen, and recent extensions to cases where the obstacle Γ is some arbitrary compact set, including cases where Γ is fractal or has fractal boundary.

In all these cases the problem reduces to an operator equation $A\phi = g$, where $A : H \to H^*$ is some boundary integral operator from H, a Hilbert space of distributions on Γ, to its dual space H^*. In each case our goal is to bound the norm of A and its inverse, and thus bound the condition number of A. Our new results (this joint work with Siavash Sadeghi) include the first bounds for A and A^{-1} that apply for general compact Γ, and the observation that, in the class of arbitrary compact Γ, the condition number of A may increase arbitrarily fast as the wavenumber $k \to \infty$ through some sequence, even if the complement of Γ is connected.